A.J. SADLER

MATHEMATICS METHODS

UNIT 1

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A. J. Sadler

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Preface.

This text targets Unit One of the West Australian course *Mathematics Methods*, a course that is organised into four units altogether, units one and two for year eleven and units three and four for year twelve.

UNIT ONE	UNIT TWO	UNIT THREE	UNIT FOUR

The West Australian course, *Mathematics Methods*, is based on the Australian National Curriculum Senior Secondary course *Mathematical Methods*. At the time of writing there is very little difference between the content of Unit One of the West Australian course *Mathematics Methods*, and Unit One of the National Curriculum course *Mathematical Methods*, so this text would also be suitable for anyone following Unit One of the National Curriculum course, *Mathematical Methods*.

The book contains text, examples and exercises containing many carefully graded questions. A student who studies the appropriate text and relevant examples should make good progress with the exercise that follows.

The book commences with a section entitled *Preliminary work* to give the reader an early reminder of some of the work from earlier years that it will be assumed readers are familiar with, or for which the brief outline included in the section may be sufficient to bring the understanding of the concept up to the necessary level.

As students progress through the book they will encounter questions involving this preliminary work in the *Miscellaneous Exercises* that feature at the end of each chapter. These miscellaneous exercises also include questions involving work from preceding chapters to encourage the continual revision needed throughout the unit.

Students should also find that the content in some chapters involves work encountered in previous years, thus allowing speedy progress through those sections.

Some chapters commence with a "Situation" or two for students to consider, either individually or as a group. In this way students are encouraged to think and discuss a situation, which they are able to tackle using their existing knowledge, but which acts as a fore-runner and stimulus for the ideas that follow. Students should be encouraged to discuss their solutions and answers to these situations and perhaps to present their method of solution to others. For this reason answers to these situations are generally not included in the book.

At times in this series of books I have found it appropriate to go a little outside the confines of the syllabus for the unit involved. In this regard readers will find in this text that the Pythagorean theorem is assumed and used, the distance between two points with known coordinates is covered and the trigonometric identity $\sin^2 A + \cos^2 A = 1$ is included. When considering $y = a \sin x$, $y = \sin bx$ and $y = \sin (x - c)$ I also consider the more general $y = a \sin [b(x - c)] + d$.

The sine and cosine rules are considered early in this text so that students studying *Mathematics Specialist* cover this work in good time for its use in that course.

Alan Sadler

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Important note.

This series of texts has been written based on my interpretation of the appropriate *Mathematics Methods* syllabus documents as they stand at the time of writing. It is likely that as time progresses some points of interpretation will become clarified and perhaps even some changes could be made to the original syllabus. I urge teachers of the *Mathematics Methods* course, and students following the course, to check with the appropriate curriculum authority to make themselves aware of the latest version of the syllabus current at the time they are studying the course.

Alan Sadler